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DECAY TURTLE

(Trace Unlimited Rays Through Lumped Elements)

A COMPUTER PROGRAM FOR SIMULATING CHARGED PARTICLE BEAM TRANSPORT SYSTEMS, INCLUDING DECAY CALCULATIONS

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1. INTRODUCTION

TURTLE is a computer program designed to simulate charged particle beam transport systems. It allows evaluation of the effect of aberrations which exist in beams with small phase-space volume. These include higher-order chromatic aberrations, effects of non-linearities in magnetic fields, and higher-order geometric aberrations due to the accumulation of second-order effects. The beam at any point in the system may be represented by one- and two-dimensional histograms.

The original version of TURTLE was created by D.C. Carey at NAL*). The major part of this program and of the present manual are his work.

In the CERN version of TURTLE, hereafter called DECAY TURTLE, provision has been made to include particle decay, following the parent particles and up to two kinds of daughter particles through the beam line. New additions to the code for this CERN version are indicated by italic script to distinguish from original TURTLE as written by D.C. Carey.

*) TURTLE (Trace Unlimited Rays Through Lumped Elements), A computer program for simulating charged particle beam transport systems, by D.C. Carey (National Accelerator Laboratory, Batavia, IL 60510, USA, Report NAL-64, December 1971).

2. GENERAL THEORY

2.1 Uses and limitations of the matrix approach

As in TRANSPORT, we represent the position and direction of travel of a particle entering a beam line via a vector with six coordinates[1,2]

$$X = \begin{bmatrix} x \\ x' \\ y \\ y' \\ \ell \\ \delta \end{bmatrix} \tag{1}$$

The coordinates x and y represent, respectively, the horizontal and vertical displacements at the position of the particle, and x' and y' represent the angles with the axis of the beam line in the same planes. The quantity ℓ represents the longitudinal position of the particle relative to a particle travelling on the magnetic axis of the system with the central momentum designed for the system. The remaining quantity $\delta = (p - p_0)/p_0$ gives the fractional deviation of the momentum of the particle from the central design momentum of the system.

The effect of the passage of a particle across a magnetic element or a drift space may be represented to first order by a transfer matrix R. The coordinates X(1) of the particle at the end of the element are then given in terms of those at the beginning X(0) as

$$X(1) = RX(0) \ . \tag{2}$$

The effect of successive elements, each with its own R matrix may be given by a total R matrix R(t) which is equal to the product of the individual R matrices.

$$R(t) = R_n R_{n-1} \cdots R_2 R_1 \ . \tag{3}$$

The first columns of such a matrix are obtained by solving a differential equation of the form

$$g'' + k^2 g = 0 \tag{4}$$

with appropriate boundary conditions, and taking either the value or the derivative of the solution. The differentiation is with respect to distance along the beam line and k^2 is a function only of that quantity. The last two columns are obtained by solving the equation

$$g'' + k^2 g = f \ , \tag{5}$$

where f is a driving term given in terms of quantities which depend only on the position along the beam line and single factors of solutions to either Eq. (4) or (5). Equation (5) may be solved by a Green's function by writing

$$g(t) = \int_0^t G(t,\tau) f(\tau)\, d\tau \ . \tag{6}$$

If we treat Eq. (2) as the first term of a power series expansion of X(1) in terms of X(0) we may write further terms as follows:

$$x_i(t) = \sum_j R_{ij} x_j(0) + \sum_{j,k} T_{ijk} x_j(0) x_k(0) \ . \tag{7}$$

Here our definition of the T matrix differs from Brown's[1] in that ours is rectangular while Brown's is triangular. The use of a rectangular matrix is more easily extended to the study of higher-order effects. The difference between the two matrices is that the off-diagonal elements of the rectangular matrix are exactly half those of the triangular matrix.

As in the case of the R matrix, the T matrix, for a succession of elements, may be written in terms of the R and T matrices of the individual elements via a product. The T matrix T(t) for a succession of two elements is written in terms of the individual R and T matrices as follows:

$$T(t)_{ijk} = \sum_\ell R^{(2)}_{i\ell} T^{(1)}_{\ell jk} + \sum_{\ell m} T^{(2)}_{i\ell m} R^{(1)}_{\ell j} R^{(1)}_{mk} \ . \tag{8}$$

Notice that at no point in the evaluation of a T matrix for a beam line are two individual T matrices multiplied together. Such a multiplication would yield terms of order higher than second. Since in a matrix approach one characterizes the effects of the beam line according

to order, and we are here expanding to second order, we must consistently truncate our results to second order.

In a matrix approach the beam itself may also be represented by a matrix σ. The phase space occupied by the beam is taken to be an ellipsoid in six dimensions. Usually this matrix is initially taken to be diagonal with the square roots of the diagonal elements being equal to the semi-axes of the ellipsoid. The beam ellipsoid at a later point is obtained, using the R matrix, as

$$\sigma_1 = R\sigma_0 R^T \ . \tag{9}$$

The diagonal elements are now the squares of half the maximum extent of the ellipsoid in a given dimension. The off-diagonal elements give the correlation between the coordinates, as for a tilted ellipse. A phase-space envelope which is initially ellipsoidal continues to be so only when we limit our consideration to first-order effects. When higher orders are considered the elements of the beam matrix may be taken as second moments of a distribution, but give us no information as to the actual shape of the phase-space envelope.

The matrix method is indispensable for fitting and is the only reasonable way to obtain an initial design of a beam. It is also unsurpassed as a tool to determine individual second-order influences on particle trajectories and minimizing such aberrations. Its limitations are in the representation of the beam phase space and the fact that the results are limited by the order of the Taylor's series expansion.

2.2 Ray-tracing methods

A true ray-tracing program computes the trajectory of a particle through a magnetic field directly. It, therefore, does not distinguish among different order effects on a ray, but represents all orders to the accuracy of the numerical integration of the equations of motion. Such an approach is also useful for determining individual contributions to aberrations. If, however, one wishes to represent the phase space occupied by the beam, it is necessary to run large numbers of rays through the system to obtain a reasonable population. For a complicated system this approach can prove time consuming. In addition it is often unnecessary, as in the case of beams possessing a small phase-space volume.

2.3 Theory of ray tracing through lumped elements

In order to describe the procedure employed in TURTLE, we must explain what is meant by the distinction between local and global classification of aberrations according to order. If we once again let X be a vector giving the coordinates and direction of a particle (henceforth referred to as a ray), and expand the differential equation of motion of a particle passing through a magnetic field in powers of X, we obtain, deleting subscripts,

$$DX + EX^2 + FX^3 + \ldots = 0 \ . \tag{10}$$

The coefficients D, E, and F are matrices, with the diagonal terms of D being differential operators

$$DX = \left(\frac{d^2}{dt^2} + k^2\right) X \ . \tag{11}$$

The first-order transfer matrix R is obtained by solving the differential equation to first order, i.e.

$$DX = 0 \tag{12}$$

The second-order transfer matrix T is now obtained from the second-order coefficients in the differential equation via a Green's function

$$T(t,0) = \int_0^t G(t,\tau)E(\tau)X^2(\tau)\, d\tau \ , \tag{13}$$

where X(t) is obtained from the first-order transformation

$$X(t) = R(t,0)X(0) \ . \tag{14}$$

We see that to second order, the coefficients of a given order in the differential equation of motion give rise to transfer matrices of the same order. If, however, we wish to go further and ask for third-order transfer matrices $U_{ijk\ell}$, then we find that

$$U = \int_0^t G(t,\tau)F(\tau)X^3(\tau)\,d\tau$$
$$+ \int_0^t G(t,\tau_1)E(\tau_1)X(\tau_1) \int_0^{\tau_1} G(\tau_1,\tau_2)E(\tau_2)X^2(\tau_2)\,d\tau_2\,d\tau_1$$
$$+ \int_0^t G(t,\tau_1)E(\tau_1) \int_0^{\tau_1} G(\tau_1,\tau_2)E(\tau_2)X^2(\tau_2)\,d\tau_2 X(\tau_1)\,d\tau_1 \ .$$

The first term contains the third-order coefficients in the differential equation, while the second and third contain the second-order coefficients. Elements of transfer matrices beyond second order therefore involve not only coefficients of the differential equation of the same order, but also accumulations of lower-order terms. We, accordingly, call a classification by order of aberrations according to their appearance in the equation of motion as a local classification. A classification of terms by appearance in a transfer matrix will be called global.

A matrix approach as defined in Section 2.1 classifies aberrations by order both locally and globally. Such an approach is necessary for any order if one is interested in correcting aberrations of that order. A true ray-tracing program as explained in Section 2.2 need not classify aberrations by order at all.

In the lumped element approach to ray tracing, we classify aberrations by order locally but not globally. The passage of a ray across an individual element is given by a transformation which yields the output ray directly from the input ray. A large number of rays can then be passed through a system in a short time. The coordinates of the rays can then be collected at any point in the beam line and histograms can be generated. We can then exhibit the phase space occupied by the beam explicitly, and do not have to depend on an ellipsoid formalism.

The use of the lumped element approach permits the inclusion of many sorts of aberrations, but effectively precludes the use of others. To understand how TURTLE works we must examine the different types of local aberrations. We characterize local aberrations into four types: chromatic effects, geometric effects, magnetic field effects, and mixtures of any of the first three types.

Chromatic effects are due to a deviation of the momentum of a ray from the central momentum of the beam line. An example would be the chromatic aberration of a quadrupole, where the focusing strength depends on the momentum of the particle. Geometric effects are associated with the phase space accepted by the beam line. A trajectory entering a bending magnet at an angle to the central trajectory traverses a different path length in the field and is therefore bent through a different angle than is the central trajectory. This effect gives rise to second- and higher-order geometric aberrations for bending magnets. Another example occurs in the normal treatment of quadrupoles. The harmonic oscillator approximation for the motion of a charged particle in a quadrupole is based on a small angle approximation. Corrections for this approximation involve geometric aberrations of third and higher order. Magnetic field effects are due to non-linearities appearing in the expansion of the magnetic field in terms of the transverse coordinates of the beam line. An example is the effect of a sextupole for on-momentum rays. Mixed effects represent combinations of the above effects. One example might be the momentum dependence of the effect of a sextupole.

The intent of creating TURTLE was to examine chromatic aberrations and the effect of non-linearities in magnetic fields to all orders, and to evaluate the effect of slits and apertures and to represent the beam phase-space distribution, including effects of second and higher orders. Geometric effects are considered locally only to second order, but higher order global effects will appear due to the accumulation of second-order effects. It was not possible to achieve this goal rigorously, so we explain below what was done in each case and why the approximations used should be valid for beams possessing a small monoenergetic phase-space volume.

Transfer matrix elements for quadrupoles and sextupoles are evaluated directly for each ray from the actual momentum of the ray, and are, therefore, exact to all orders in chromatic effects. However, in bending magnets, chromatic effects are evaluated only to second order. But in high-energy separated function beams the net focusing effects of bending magnets are small compared to quadrupole contributions, so it is usually sufficient to include only second-order chromatic corrections for bending magnets.

In decay problems, a broader momentum band of particles is generated and must be tracked through the beam line. In DECAY TURTLE we have therefore modified the treatment of bending magnets to improve the precision of calculation of the chromatic (momentum dependent) terms.

Because of the small phase-space volume occupied by typical high-energy beams, the third and higher order geometric effects of a given element will be small. However, higher-order global terms may occur due to cumulative effects of second-order terms. The large lever arms for aberrations in beams at high energies will enhance the importance of such cumulative effects relative to local higher-order geometric effects.

The inclusion of apertures and slits and the representation of the beam phase space with histograms allow a further step in the realistic representation of a beam.

Below we explain how to use the program and further describe each element available.

3. USE OF TURTLE

TURTLE is designed to be run using the same deck of data cards as was used for TRANSPORT[3]. Typically one will start with a deck of cards containing approximate values of the final parameters to use in obtaining a fit to the desired constraints. Once the final set of parameters is known, one alters this deck to include them for purposes of studying second-order aberrations and misalignments. With a few changes this deck may be used as input for TURTLE. The data format is free field and the deck structure is the same. Below we discuss the deck structure and each of the type codes. For completeness we include many things that are unchanged from TRANSPORT.

3.1 Structure of the deck

1) Title card

The first card of the deck contains the title of the run enclosed in single quotes. No other item should appear on this card.

2) Number of rays

The second card in a TRANSPORT deck contains an integer which serves as an indicator. When using TURTLE this card contains an integer indicating how many rays one wishes to run through the system. Naturally one will want to choose this number sufficiently high to obtain good statistics. A number of rays equal to several thousand should be quite reasonable for any beam. The only limitation on the number of rays that can be run will be the use of computer time. The user will eventually wish to select this number on the basis of his own experience.

3) Elements

The elements with their type codes and appropriate parameters and labels are entered in sequence just as in TRANSPORT. Each element must be followed by a semicolon. The labels are not used by TURTLE but may be retained and will appear in the output. This will enable the user to compare his output from TURTLE with that from TRANSPORT. There are additional type codes indicating the creation of histograms. All type codes, including those for histograms, will be explained below.

4) Comments

Comments may be placed in the data deck before any type code entry. They are indicated by enclosing in parentheses.

5) Sentinel

The input data is terminated with a SENTINEL card. The program, after reading and initially processing the data, runs the specified number of rays through the beam line and collects and plots histograms.

As in TRANSPORT, several problem decks may be stacked together and run in the same job. However, the problems cannot depend on each other, as is possible in a TRANSPORT run via the use of the ±1 indicator card. Instead each problem deck must specify a complete beam line.

Table 1: Summary of DECAY TURTLE type codes

PHYSICAL ELEMENT	TYPE CODE	2nd ENTRY	3rd ENTRY	4th ENTRY	5th ENTRY	6th ENTRY	7th ENTRY	8th ENTRY	9th ENTRY
BEAM	1.0	x (cm)	x′ (mr)	y (cm)	y′ (mr)	ℓ (cm)	δ (percent)	p_o	
r.m.s. ADDITION TO BEAM ENVELOPE	1.0	Δx (cm)	Δx′ (mr)	Δy (cm)	Δy′ (mr)	Δℓ (cm)	Δδ (percent) where $\delta = (p - p_o)/p_o$	Δp_o (GeV/c)	id
POLE-FACE ROTATION (for bending magnets)	2.0	ANGLE OF ROTATION (degrees)							
DRIFT	3.0	LENGTH (metres)							
BENDING MAGNET	4.0	LENGTH (metres)	FIELD (kG)	FIELD GRADIENT (n-value)					
QUADRUPOLE	5.0	LENGTH (metres)	FIELD (kG)	HALF-APERTURE (cm)					
SLIT	6.0	id + i	HALF WIDTH (cm)						
ELLIPTIC SLIT	6.0	id + i	HALF AXIS (cm)	j	HALF AXIS (cm)				
BEAM CENTROID SHIFT	7.0	SHIFT (x)(cm)	SHIFT (x′)(mr)	SHIFT (y)(cm)	SHIFT (y′)(mr)	SHIFT (ℓ)(cm)	SHIFT (δ percent)		
REPEAT CONTROL	9.0	NUMBER OF REPEATS	(The end of repeat section is signified by a (9. 0. ;) entry)						
APERTURE CONSTRAINT	13.0	id + 10							
ARBITRARY R MATRIX	14.0	R(J,1)	R(J,2)	R(J,3)	R(J,4)	R(J,5)	R(J,6)	J	
UNITS CONTROL (transport dimensions)	15.0	CODE	LABEL	SCALE FACTOR (if required)					
QUADRATIC TERM OF BENDING FIELD	16.0	1	$\epsilon(1) = \beta\left(\frac{1}{\rho_o}\right)^2$	ρ_o in units of transverse length (cm)					
HALF-APERTURE OF BENDING MAG. in x-PLANE	16.0	4	w/2 (cm)						
HALF-APERTURE OF BENDING MAG. in y-PLANE (gap)	16.0	5	g/2 (cm)						
FRINGING FIELD CORRECTION COEFFICIENT	16.0	7	K_1 (dimensionless)						
FRINGING FIELD CORRECTION COEFFICIENT	16.0	8	K_2 (dimensionless)						
FORCE DECAY LENGTH	16.0	9	Z_{max} (metres)						
DESIGN MOMENTUM OF OPTIC AXIS	16.0	11	p_o (GeV/c)						
CURVATURE OF ENTRANCE FACE OF BENDING MAGNET	16.0	12	$(1/R_1)$ (1/metres)						
CURVATURE OF									

Once introduced, a 16.0 type code entry applies to all appropriate succeeding elements in a beam line unless reset to zero or a new value.

APERTURES	16.0	100		WIDTH (cm)		
HYPERBOLIC QUAD. APERTURES	16.0	101	HALF-APERTURE (RADIUS) (cm)			
PARENT PARTICLE NAME	16.0	200	"Name" (max. 4 char's)			
CHARGED DAUGHTER NAME	16.0	201	"NAME"			
NEUTRAL DAUGHTER NAME	16.0	202	"NAME"			
MULTIPOLE COMPONENTS IN QUAD.	16.0	-½ (NUMBER OF POLES)	AMPLITUDE (Normalized to quadrupole)	PHASE ANGLE (degrees)		
SECOND-ORDER CALCULATIONS	17.0					
SEXTUPOLE	18.0	LENGTH (metres)	FIELD (kG)	HALF-APERTURE (cm)		
SOLENOID	19.0	LENGTH (metres)	FIELD (kG)			
BEAM ROTATION	20.0	ANGLE OF ROTATION (degrees)				
ONE-DIMENSIONAL HISTOGRAM	50.n	id + i	LOWER LIMIT	UPPER LIMIT	BIN SIZE	
TWO-DIMENSIONAL HISTOGRAM (x-COORDINATE)	51.n	id + i	LOWER LIMIT	UPPER LIMIT	BIN SIZE	
TWO-DIMENSIONAL HISTOGRAM (y-COORDINATE)	52.n	id + i	LOWER LIMIT	UPPER LIMIT	BIN SIZE	
HISTOGRAM FLAG	53.n	(see text for explanation)				
DECAY	60.0	MASS OF PARENTS (GeV)	MASS OF CHARGED DAUGHTERS (GeV)	MASS OF NEUTRAL DAUGHTERS (GeV)	PARENT LIFETIME (nanoseconds)	*Both first- and second-order calculations are implemented by type code 60.0.*

All type code entries must be terminated by a semicolon (;) to indicate their end, e.g. 2. 20. ;

id stands for particle identification, where

id = 0 signifies parents
id = 100 signifies charged daughters
id = 200 signifies neutral daughters

r.m.s. Addition to Beam

An r.m.s. addition to beam

(1 Δx $\Delta x'$ Δy $\Delta y'$ $\Delta \ell$ $\Delta \frac{\Delta p}{p}$ Δp id ;)

must specify the particles to be affected via

id = 0 parents
id = 100 charged daughters
id = 200 neutral daughters

It will affect all particles of the specified kind unless it is immediately preceded by a slit with the same particle id, in which case it will affect only the particles outside the specified slit aperture.

i,j stand for coordinates, as follows

i,j	Coordinate	Standard units
1	x	cm
2	x'	mr
3	y	cm
4	y'	mr
6	$\delta = \frac{p - p_0}{p_0}$	%
8	z (longitudinal length)	m
11	p (particle momentum)	GeV/c
21	$r = \sqrt{x^2 + y^2}$ (polar radius)	cm
22	$\zeta = \sqrt{x'^2 + y'^2}$ (emittance angle)	mr
23	ψ = atan y/x (polar angle)	degrees
24	ζ = atan y'/x'	degrees

3.2 Type codes

BEAM: Type code 1.0

Type code 1.0 can be used either to represent the initial phase space of the beam or a random addition to the beam. There are either eight or nine parameters on the beam card, as follows:

1 - Type code 1.0

2 - One half the horizontal extent x of the beam (cm in standard units).

3 - One half the horizontal angular divergence x′ (mr in standard units).

4 - One half the vertical extent y of the beam (cm in standard units).

5 - One half the vertical angular divergence y′ (mr in standard units).

6 - One half the longitudinal extent of the beam. In TURTLE this coordinate is not affected as the ray proceeds down the beam line.

7 - One half the momentum spread δ of the beam (in units of percent)

$$\delta = \frac{P - P_0}{P_0} = \frac{\Delta p}{P_0} .$$

8 - The central design momentum of the beam (GeV/c in standard units).

9 - The code digit *id* indicating a random addition to the beam. If one is specifying the initial phase space this entry must be absent.

In TRANSPORT and original TURTLE, the code digit id for a random addition to the beam is always zero. However in DECAY TURTLE, id must identify the particle to which the random addition is to be made as follows:

id = 0 adds a random addition to the parent particles only.
id = 100 " " " " to the charged daughters only.
id = 200 " " " " to the neutral daughter only.

Each ray is chosen at random with its coordinates constrained to lie within the limits specified on the beam card. In addition, the x and y

coordinates are constrained to lie within an upright ellipse whose semi-axes are the quantities given on the beam card. The angular coordinates x' and y' are also constrained to lie within a similar ellipse. This prevents the implicit representation of a preferred direction in space by the choice of coordinates. In other words, targets may be circular and scattering is isotropic. Further restrictions on the phase space, such as the use of a multi-dimensional ellipsoid were felt not to be physically realistic. This is illustrated by the fact that the target is the same size for particles of different momentum.

If we let x_0, x_0', y_0, y_0', ℓ_0 and δ_0 represent the maximum extents of the beam envelope in each coordinate, then the additional restriction imposed by TURTLE is equivalent to the two equations

$$\left(\frac{x}{x_0}\right)^2 + \left(\frac{y}{y_0}\right)^2 \leq 1 ,$$

$$\left(\frac{x'}{x_0'}\right)^2 + \left(\frac{y'}{y_0'}\right)^2 \leq 1 .$$

The hypervolume of this four-dimensional region is equal to $\pi^2 x_0 x_0' y_0 y_0'$. By contrast the hypervolume of the region specified by the single ellipsoidal condition

$$\left(\frac{x}{x_0}\right)^2 + \left(\frac{x'}{x_0'}\right)^2 + \left(\frac{y}{y_0}\right)^2 + \left(\frac{y'}{y_0'}\right)^2 \leq 1$$

is equal to $(\pi^2/2) x_0 x_0' y_0 y_0'$, or exactly one half that of the region specified by the first set of conditions. This additional volume occurs near the boundaries of the region specified in the two-ellipse condition. When the multi-dimensional distribution is projected onto one or two dimensions the ellipsoidal condition yields a distribution which is very sparse near the edges. The two-ellipse condition yields a projection where the edges of the distribution are much more sharply defined and is thus much better suited to beam line studies.

For a random addition to the beam, the coordinates are chosen in the same manner, but are then added to the coordinates of the ray at the point where the random addition is made. Because of the random selection of both the original ray and the addition to it the effective widths of the distributions in any coordinate will add in an r.m.s. fashion. It

should be remembered that the parameters on the card indicating the random addition specify the maximum amplitude of the scattering. If the element immediately preceding a random addition is a slit (type code 6.0) with the same particle id the random addition will be made only if the ray hits the slit. If the ray passes through the slit its coordinates will be unchanged and it will continue unperturbed. *Note that for original TURTLE the particle id is zero and is the same by definition on both type code entries.*

FRINGING FIELDS and POLE-FACE ROTATIONS for bending magnets: Type code 2.0

Type code 2.0 specifies the pole-face rotation of a bending magnet and causes the effect of the fringing fields to be calculated. It should immediately precede or follow a type code 4.0 representing a bending magnet. There are two parameters.

1 - Type code 2.0

2 - The pole-face rotation angle (in degrees).

Even if the pole-face rotation angle is zero, it is necessary to insert a 2.0 card to take the fringing field into account. When the program is run the effects of the fringing field and the bending magnet are included in a single transfer matrix for the entire magnet. It is therefore not possible to insert a histogram between a type code 2.0 and 4.0 element. More information about the transfer matrix will be found in the description of the 4.0 type code.

The matrix elements for pole-face rotations and fringing fields of bending magnets are calculated to second order only for both versions of TURTLE. See the section under type code 4.0 for a description of the method of calculation for the magnetic fields for the interior portion of sector bending magnets.

DRIFT SPACE: Type code 3.0

A drift space is a region containing no magnetic elements. Two parameters are required.

1 - Type code 3.0

2 - Length (normal unit is metres).

SECTOR BENDING MAGNET: Type code 4.0

For a sector bending magnet the entrance and exit faces of the magnet are straight and perpendicular to the central axis of the beam. Other types of magnets may be represented by using a type code 2.0 and any of the bending magnet parameters included under type code 16.0. A type code 4.0 requires four parameters.

1 - Type code 4.0

2 - The effective length of the central trajectory through the magnetic field (normal unit is metres).

3 - The field strength B_0 along the central trajectory (normal unit is kilogauss).

4 - The field gradient n in dimensionless units.

$$n = \frac{-\rho_0}{B_0} \frac{\partial B}{\partial x} (x,o,t)$$

where $\rho_0 = p_0/(qB_0)$, p_0 is the central momentum at the beam, q the charge of the particle, and ρ_0 the radius of curvature of the central trajectory.

If second order (see type code 17.0) is not specified, a ray with the beam central momentum is transformed through the magnet using the first-order transfer matrix R so that X(1) = RX(0). If the ray does not have the central momentum we use an off-momentum R matrix which is obtained from the ordinary first-order matrix R^0 and the chromatic elements of the second-order matrix T via the equation

$$R_{ij} = R^0_{ij} + T_{ij6} \frac{\Delta p}{p_0}$$

where $\Delta p/p_0 = (p - p_0)/p_0$ is the fractional deviation from the central momentum. If a type code 17.0 card is included in the deck, second-order geometric effects will also be included in the calculations. These correspond to the matrix elements T_{ijk}, where j and k are both equal to or less than four.

In DECAY TURTLE, a complete second-order calculation is automatically invoked when a type code 60.0 card is inserted calling for decay calculations to be made.

In DECAY TURTLE, the treatment of momentum-dependent terms in sector bending magnets differs from that in original TURTLE as follows: Consider the equations of motion as given on page 34 of Ref. 1:

$$x'' - h(1 + hx) - x'(hx' + h'x) =$$

$$= \frac{p_0}{p} T'\{(1 + hx)[-h + nh^2x - \beta h^3x^2 + \tfrac{1}{2}(h'' - nh^3 + 2\beta h^3)y^2] + h'yy' + \ldots\}$$

$$y'' - y'(hx' + h'x) =$$

$$= \frac{p_0}{p} T'\{-h'x'y - (1 + hx)[nh^2y - 2\beta h^3xy] + \ldots\} .$$

We now introduce the second-order expansion

$$T' = 1 + hx + \tfrac{1}{2}(x'^2 + y'^2) + \ldots$$

and redefine the fractional momentum change to be

$$\varepsilon = \frac{p - p_0}{p} = \frac{\delta}{1 + \delta} \quad \text{where } \delta = \left(\frac{p - p_0}{p_0}\right) \text{ from Ref. 1;}$$

then the equations of motion may be rewritten to second order as follows:

$$x'' + h^2(1 - n)x = h\varepsilon +$$
$$+ h^3(2n - \beta - 1)x^2 + h'xx' + \tfrac{1}{2}hx'^2 + h^2(2 - n)x\varepsilon +$$
$$+ \tfrac{1}{2}(h'' - nh^3 + 2\beta h^3)y^2 + h'yy' - \tfrac{1}{2}hy'^2 + \ldots$$
$$y'' + h^2ny =$$
$$= 2h^3(\beta - n)xy + h'xy' - h'x'y + hx'y' + nh^2y\varepsilon + \ldots .$$

Note that these equations have the same general form as those given in Ref. 1, except that here no ε^2 term appears in the x equations. Using the same method as in Ref. 1, we have solved these equations and find that all matrix elements remain the same, except for T_{166} and T_{266} which now have the form:

$$T_{166} = h^2(2 - n)I_{16} + h^3(2n - \beta - 1)I_{166} + \tfrac{1}{2}h^3I_{122}$$

$$T_{266} = h^2(2 - n)I_{26} + h^3(2n - \beta - 1)I_{266} + \tfrac{1}{2}h^3I_{222} - hd_xd'_x .$$

Decay TURTLE uses these modified equations, except for uniform field magnets ($n = 0$, $\beta = 0$) *in which case the following exact formula set is used to describe the trajectories:*

$$x_2 + \rho_0 = (x_1 + \rho_0) \cos \alpha + \rho[\cos \theta_2 - \cos (\theta_1 + \alpha)]$$

$$\rho \sin \theta_2 = \rho \sin (\theta_1 + \alpha) - (x_1 + \rho_0) \sin \alpha$$

$$y_2 = y_1 + y_1' \rho(\alpha + \theta_1 - \theta_2) \cos \theta_1$$

$$y_2' = y_1' \frac{\cos \theta_1}{\cos \theta_2} .$$

where:

ρ_0 *is the radius of curvature of the central trajectory having a momentum* p_0.

ρ *is the radius of curvature of an arbitrary trajectory having a momentum* p.

α *is the total bending angle of the central trajectory.*

θ_1 *is the entrance angle made between the central trajectory and the projection of an arbitrary trajectory onto the bend (mid-) plane of the magnet.*

θ_2 *is the exit angle made between the central trajectory and the projection of an arbitrary trajectory onto the bend (mid-) plane of the magnet.*

y_1' *is the tangent of the angle* ϕ_1 *made between the central trajectory and the projection of an arbitrary trajectory onto a plane perpendicular to the midplane, at the entrance of the magnet (see Fig. 1).*

y_2' *is the tangent of the angle* ϕ_2 *made between the central trajectory and the projection of an arbitrary trajectory onto a plane perpendicular to the midplane, at the exit of the magnet (see Fig. 1).*

x_1, y_1, x_2, y_2 *are the x and y coordinates of the arbitrary trajectory at the entrance and exit planes of the magnet relative to the central trajectory.*

This set of exact equations is valid for uniform field sector (wedge) bending magnets (ignoring entrance and exit fringing fields) and can be verified using the constructions in Figs. 1 and 2.

No fringing-field effects or pole-face rotations are included in type code 4.0 entries. To include them type code 2.0 cards must be inserted before and after such a magnet, even if the pole-face rotation angle is zero.

Original TURTLE assembles a single transfer matrix for each bending magnet including the fringing fields. It then transforms each ray using this assembled transformation matrix. In DECAY TURTLE, the tracking through a bending magnet normally consists of three steps:

- *tracking through the entrance fringing field, if present,*
- *tracking through the idealized sector bending magnet, and*
- *tracking through the exit fringing field, if present.*

This three-step process has been used because, if a decay occurs within the magnet, the transform must be split correctly at the point of decay.

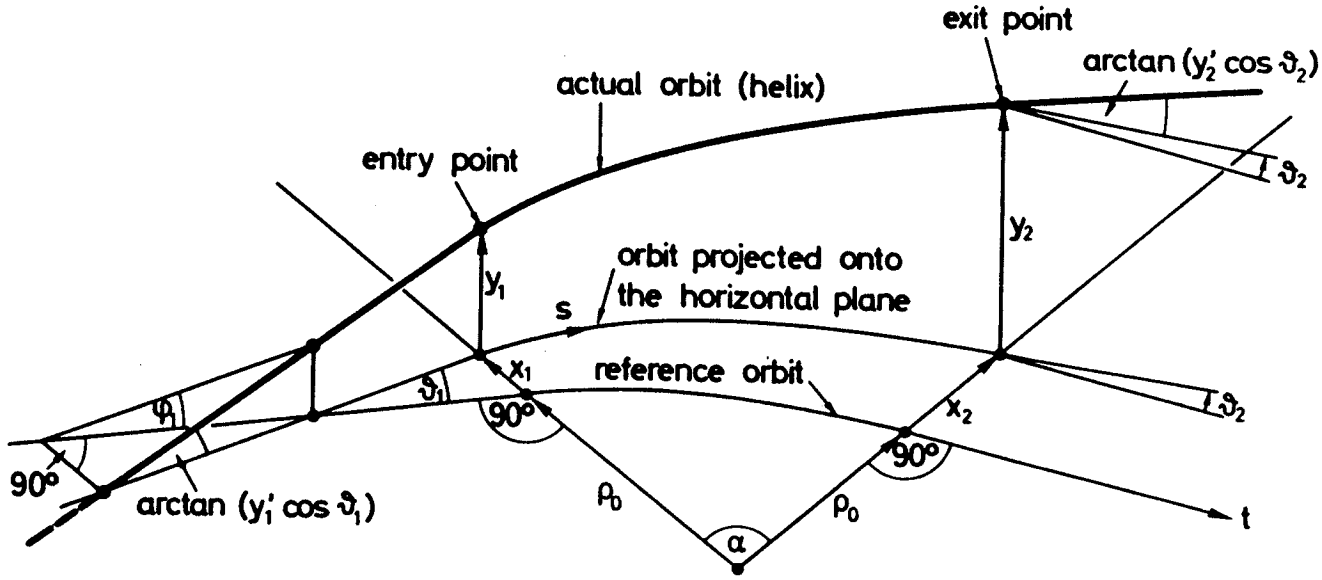


Fig. 1 View of the three-dimensional orbit in a homogeneous field sector bending magnet

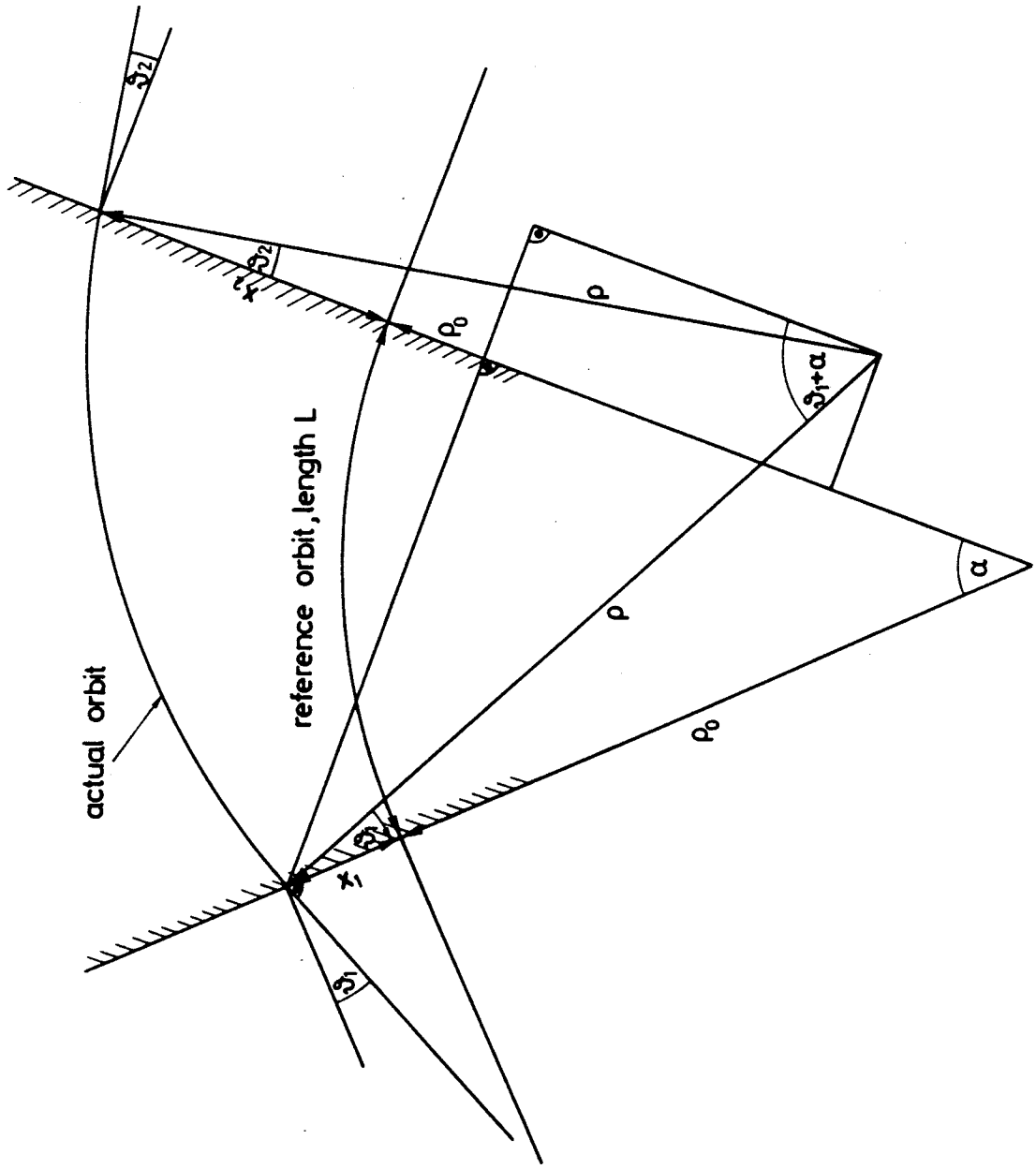


Fig. 2 Projection of Fig. 1 onto the magnetic midplane

QUADRUPOLE: Type code 5.0

A quadrupole requires four parameters for its specification.

1 - Type code 5.0

2 - The effective length of the quadrupole field (normal unit is metres).

3 - The pole tip field, B_0, a positive field indicating a horizontally-focusing quadrupole (normal unit is kilogauss).

4 - Pole tip half aperture a. The gradient is obtained by dividing the pole tip field by this aperture.

For a perfect quadrupole the first four components of the ray vector $X = (x,x',y,y',\ell,\delta)$ are carried through the quadrupole via the transformation matrix

$$\begin{pmatrix} \cos k\ell & \frac{1}{k}\sin k\ell & 0 & 0 \\ -k\sin k\ell & \cos k\ell & 0 & 0 \\ 0 & 0 & \cosh k\ell & \frac{1}{k}\sinh k\ell \\ 0 & 0 & k\sinh k\ell & \cosh k\ell \end{pmatrix}$$

where $k^2 = qB_0/ap$, p is the actual momentum of the ray, and q is the charge of the particle. This matrix is for a horizontally-focusing quadrupole. A matrix for a vertically-focusing quadrupole is obtained by exchanging the two submatrices occurring on the diagonal. Since this matrix is evaluated for each ray using the actual momentum of the ray it is valid to all orders in chromatic effects.

Higher-order multipole effects for a realistic quadrupole may also be introduced via a 16.0 type code entry. Multipoles up to and including a 40-pole may be included. In original TURTLE the multipole aberrations are taken to be lumped at the longitudinal midpoint of the quadrupole. The ray is transformed half-way through the quadrupole, perturbed by the multipole aberrations, then transformed through the remainder of the quadrupole. If no multipole aberrations are included the ray passes through the quadrupole in a single step. The multipole component strengths are normalized to unit gradient and need be entered only once

for similar but differently excited quadrupoles. The effect of the multipoles is also calculated from the actual momentum of the ray, making this effect correct to all orders chromatically.

While the quadrupole matrix elements are computed in the same way in both versions of TURTLE, DECAY TURTLE applies the multipole aberrations half at the entrance and half at the exit of the quadrupole rather than at its centre. However, this does not change the coding procedure for the user.

SLIT: Type code 6.0

A slit entry requires three parameters:

1 - Type code 6.0

2 - The code number *i + id* for the coordinate and the particle identification (see below).

3 - The half opening of the slit (in the units for the appropriate coordinate).

In DECAY TURTLE, an elliptic slit is available using five parameters:

1 - Type code 6.0

2 - The code number i + id for the first coordinate and the particle identification (see below).

3 - The first half axis of the ellipse.

4 - The code number j for the second coordinate. (Any particle identification on this entry is ignored.)

5 - The second half axis of the ellipse.

The code digit *i* or *j* specifies the coordinate as follows:

1 - x coordinate

2 - x′ coordinate

3 - y coordinate

4 - y′ coordinate

6 - $\delta = (p - p_0)/p_0$.

In original TURTLE the particle identification code *id* is always zero. *In DECAY TURTLE meaningful values are:*

0 - parent particles

100 - charged daughter particles

200 - neutral daughter particles.

If a ray encounters a slit and the ray coordinates lie outside the specified aperture, the ray will be stopped. If a random addition to the beam with the same particle *id* immediately follows the slit, the particle

will not be stopped, but its coordinates will be changed by an amount within the range specified for the random addition. If the ray passes within the specified aperture of the slit, it will be left undisturbed. Note that since in original TURTLE the particle *id* is always zero on both entries, the random addition will always be associated with the slit as described above.

Example No. 1. A non-symmetric slit, aperture $-60 \leq x \leq 100$

```
7.   20.   0.   0.   0.   0.   0.   ;
6.    1.   80.  ;
7.  -20.   0.   0.   0.   0.   0.   ;
```

The two coordinate shifts will cause all particles to be temporarily shifted towards positive x's such that the beam centroid passes through the centre of the slit of half aperture 80.

Example No. 2. A slit with a window

```
6.    1.   50.  ;     aperture for parents, no scattering.
6.  101.   50.  ;
1.   ...  100.  ;     aperture for charged daughters with
                      scattering on slit.
1.   ...    0.  ;   } causes all remaining parents to
                      scatter in the window.
```

In this above example the neutral daughters are not affected by any of the slit or r.m.s. type code entries.

Example No. 3. A typical neutrino decay channel

```
 :                    } decay channel.
6.    1.    0.  ;     black beam stopper for all parent particles.
1.   ...  100.  ;     all muons are scattered in the beam stopper.
 :                    } muon channel.
6.  101.    0.  ;     black beam stopper for all muons.
3.   ...        ;     neutrino channel.
6.  201.  200.   3.  150.  ;  } neutrinos are counted only if
                              they hit an elliptic detector
                              having semi-major axes of 200 cm
                              by 150 cm in x and y directions
                              respectively.
```

SHIFT IN THE BEAM: Type code 7.0

The phase-space coordinates of all rays may be shifted by a constant amount (independent of the ray) at a given point in the beam. Seven parameters are needed.

1 - Type code 7.0.

2 to 7 - The amount of the shift in each coordinate. Units are the units used for that coordinate. Coordinate No. 5 is not used but included for compatibility with TRANSPORT.

As opposed to TRANSPORT, any volume of phase space is permitted with the use of this element. Note that the misalignment (type code 8.0) is not available in TURTLE. But a known misalignment may be simulated by sandwiching an element between two 7.0 cards depicting the misalignment encountered.

REPEAT: Type code 9.0

A section of the beam may be repeated as many times as desired by sandwiching that section between two repeat cards. The two parameters needed are:

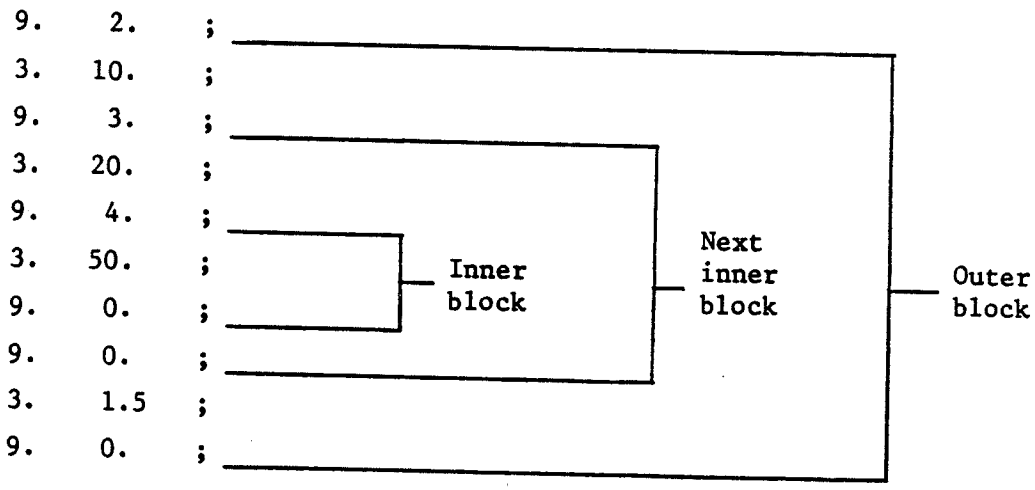
1 - Type code 9.0

2 - No. of times section is repeated. Equal to zero for end of a repeated section.

Repeats may be nested four deep. Care should be taken to ensure that for each card beginning a repeated section there is one terminating that section.

Note that histograms are not permitted within a repeat section in the present versions of TURTLE.

Example of Nesting:



The total length of this sequence is:

2*(10. + 3*(20 + 4* 50) + 1.5) = 1343.

APERTURE CONSTRAINTS: Type code 13.0

To cause the program to observe magnet apertures two parameters are required

1 - Type code 13.0

2 - The code number *id* + 10, where *id* identifies the kind of particle.

The code value *id* is always zero for original TURTLE. *In DECAY TURTLE, meaningful values for id are:*

0 - apertures observed for parent particles

100 - apertures observed for charged daughter particles

200 - apertures observed for neutral daughter particles.

If the 13. *id* + 10. ; card is not present for a given kind of particles, the only apertures affecting these particles are slits. If it is present, the program will terminate all those rays which pass outside the specified apertures of quadrupoles or bending magnets. The quadrupole half aperture is normally taken to be circular and equal to the fourth entry on the type code 5.0 card. One can specify a different aperture for quadrupoles, and make it either elliptical or hyperbolic through the use of a 16. 100. or 16. 101. element. The apertures for bending magnets are given by the 16. 4. and 16. 5. elements. All aperture constraints are applied both at the beginning and the end of the element to which they apply. See the section describing the use of the 16.0 element for further details on aperture specification.

All type code 13.0 entries normally used in TRANSPORT and not mentioned above will be ignored by TURTLE.

ARBITRARY TRANSFORMATION MATRIX: Type code 14.0

An arbitrary first-order 6 × 6 matrix transformation may be introduced. There are eight parameters.

1 - Type code 14.0

2 to 7 - The elements of the given row of the transfer matrix. The units of these numbers must be chosen to be consistent with the units for the coordinates.

8 - The number of the row.

A matrix must be read in one row at a time. An uninterrupted sequence of 14.0 type cards is taken as providing entries to the same matrix. Any unspecified row is taken to be that of the identity matrix. If two successive sets of type code 14.0 cards are to provide elements of successive matrices, they must be separated by a do-nothing entry. An example of such an element is a (3. 0. ;), a drift space of zero length.

Second-order terms may be introduced by including the 22 additional numbers.

9 - Continuation code 0.

10 to 30 - The 21 second-order matrix elements

T(i11) T(i12) T(i13) T(i14) T(i15) T(i16) T(i22)
T(i23) T(i24) T(i25) T(i26) T(i33) T(i34) T(i35)
T(i36) T(i44) T(i45) T(i46) T(i55) T(i56) T(i66)

in the order given. The letter "i" indicates the row number, and specifies the coordinate to which these matrix elements contribute.

Each set of second-order coefficients accompanies the row of the first-order matrix which contributes to the same coordinate. Once again successive 14.0 elements are taken as giving entries into the same R and T matrices. Any unspecified row is taken as being the same as the identity transformation. This means that the R matrix elements are given by the kronecker delta and the T matrix elements are all zero.

In decay TURTLE, the arbitrary transform matrix (type code 14.0) should not be used in a region of the beam line where the decay process takes place or an incorrect yield for decay particles will result.

UNITS CHANGES: Type code 15.0

Units may be changed to any desired. A specification of units change should appear before any other type code in the deck. All subsequent input data should then be consistent with the units used. There are four entries on a 15.0 card.

1 - Type code 15.0

2 - A code digit indicating which unit is to be changed.

3 - The name of the new unit. This should be enclosed in single quotes and can be at most four characters long.

4 - The size of the unit being introduced in terms of the normal unit otherwise used by the program.

Below is a table of the code digits for the units, the quantity to which they apply, and the standard unit used. The various units that may be changed are:

Code Digit	Quantity	Standard TRANSPORT Unit	Symbols used in text
1.0	horizontal and vertical transverse dimensions, and magnet apertures	cm	x,y
2.0	horizontal and vertical angles	mr	x′,y′
6.0	momentum spread	percent (PC)	δ
8.0	length (longitudinal) of elements and bending magnet pole face curvatures	metres (M)	ℓ
9.0	magnetic fields	kG	B
11.0	momentum and masses of particles used in decay process	GeV/c GeV for masses	p(0)

In certain cases the conversion factor may be omitted and the program will make the required units change by recognizing the symbol used to represent the unit. The automatic units changes available are the same as those in TRANSPORT and the reader is referred to the TRANSPORT manual for a complete listing of them.

MULTIPOLE ABERRATIONS IN QUADRUPOLES AND OTHER SPECIAL PARAMETERS: Type code 16.0

A number of parameters other than those described so far may be used to give further information about quadrupoles and bending magnets. A parameter introduced on a 16.0 card applies to all succeeding elements. It may be changed or reset to zero by introducing another 16.0 type card. Four parameters may be supplied.

1 - Type code 16.0

2 - Code digit indicating nature of parameter(s).

3 - First special parameter.

4 - Second special parameter (if needed).

Below we describe each of the special parameters available. Some of them are the same as those available in the TRANSPORT program and we refer the reader to that manual for further discussion[3].

Coding for multipole aberrations

-N. A negative code digit indicates a multipole aberration in a quadrupole. Code digit -N indicates a 2N-pole. Multipoles up to and including a 40-pole may be included. The first special parameter is equal to B_N/ga^{N-1}, where B_N is the pole-tip field due to the 2N-pole, a is the half aperture, and g is the unaberrated gradient of the quadrupole. The second special parameter is the phase angle α_N of the multipole. The spatial dependence of the magnetic potential giving rise to a multipole is then given by[4] $r^N \sin(N\theta - \alpha_N)$. Since the multipole is normalized to the gradient it may be introduced once and allowed to apply to all succeeding quadrupoles, independent of excitation.

Coding for special parameters

1. $\varepsilon(1)$ - A measure of the second-order variation with x of the magnetic field of a bending magnet. It is equal to the error in field due to this component evaluated at one horizontal unit from the beam axis, divided by the central field value, or $\Delta B_2/B_0$ at x = 1. Normally this parameter is set equal to zero.

In order for it to produce an effect, a second-order calculation must be specified by a 17.0 card. Otherwise, it will be taken to be equal to zero even if a non-zero value is specified.

5. g/2 - the gap width of a bending magnet. It is used for calculating the effect of the finite extent of the fringing field of a bending magnet, but if it is non-zero, it is also taken for the vertical half-aperture of all subsequent bending magnets. Normally it is taken to be zero.

7-8. K_1 and K_2 - Values of dimensionless integrals describing the fringing-field behaviour. Normally K_1, indicated by index code 7, is taken to be 0.5, and has a first-order effect. The quantity K_2, indicated by index code 8, is normally zero, and is felt only in second order.

9. Z_{max} - *If a type code 16. 9. Z_{max} is present in the beam line, DECAY TURTLE will track only those parent particles which decay in the portion of the beam line from*

$$0 \leq \ell \leq Z_{max} .$$

If this element is not present, it will use the real physical decay distribution. The use of this element causes all particles to decay within the channel, thus improving the statistics for daughter particles. The enhancement factor is printed by the program.

11. p_0 - *Design momentum. The type code 16. 11. p_0 ; permits the design momentum of the beam line to be redefined at any point. Its only effect is to change the radius of curvature of the reference orbit through bending magnets to correspond to a momentum p_0.*

12-13. 1/R1 and 1/R2 - The reciprocals of the radii of curvature of the entrance and exit faces, respectively, of a bending magnet. A positive value indicates the curvature is convex. Normally they are taken to be zero, meaning that the faces of the magnet are flat. They have an effect only if a second-order card, 17.0, is inserted.

Coding for magnet apertures

The magnet apertures explained below are effective only if a 13. 10. ; card is present in the beam. *In decay TURTLE the 13. 10. ; card activates the apertures only for the parent particles. For the charged daughters a 13. 110. ; must be used and for the neutral daughters a 13. 210. ; card.*

4-5. w/2 and g/2 - The horizontal and vertical half apertures, respectively, of a bending magnet. The gap width g is also used in the calculation of the fringing-field effects. Both are normally taken to be zero. If either is non-zero and the apertures are activated, they will act as an aperture stop. A ray will be stopped if it does not go through the aperture specified.

100-101. Quadrupole apertures. Normally, if aperture constraints are turned on by a 13. 10. ; entry *(or 13. 110. ; or 13. 210. ;)*, the quadrupoles are assumed to have circular apertures with a radius taken from the type code 5.0 cards. However, if a 16. 100. or 16. 101. type code occurs in a beam line, the circular aperture is deactivated for all subsequent quadrupoles and cannot be reactivated. Instead, the program will use the aperture specified on the 16. 100. or 16. 101. card. The type code 16. 100. a. b. ; specifies an elliptic aperture having the horizontal and vertical semi-axes a and b, respectively. If a circular aperture is desired, a and b must be made equal. The type code 16. 101. a ; specifies a hyperbolic aperture, where a is the radius of a circle tangent to the hyperbola

$$|xy| \leq a^2/2 \ .$$

Both elliptic and hyperbolic apertures may be specified simultaneously for the same magnet, but both will act as an aperture constraint only if apertures are activated by the 13. 10. ; type code.

Coding for particle names:

The type codes described below allow the names of the particles to be entered. The code digits are:

200 - for parent particles

201 - for charged daughter particles

202 - for neutral daughter particles.

The particle name is inserted after the code digit and must be enclosed within quotes. The name must not be longer than four characters. If a particle name is not given, DECAY TURTLE uses the name "RAYS".

Examples:

```
16.     200     "PI"    ;
16.     201     "MU"    ;
16.     202     "NU"    ;
```

SECOND-ORDER GEOMETRIC AND MAGNETIC EFFECTS: Type code 17.0

The insertion of a type code 17.0 indicates that second-order geometric and magnetic field effects are taken into consideration. All chromatic effects discussed are taken into account whenever $\delta = \Delta p/p_0$ is not zero. Also, all quadrupole multipole moments are effective whenever specified. A 17.0 card is necessary to cause second-order geometric transfer matrix elements or second-order field variations in a bending magnet to have an effect. It is also necessary in order for the magnetic field of a sextupole to have an effect. Without a 17.0 card a sextupole is taken to be a drift space.

In decay TURTLE, a second-order calculation is automatically invoked when a type code 60.0 card is present in the data deck calling for a decay calculation.

SEXTUPOLE: Type code 18.0

A sextupole may be inserted for correction of second-order aberrations. Unless a 17.0 card is included it acts like a drift space. Four parameters are needed.

1 - Type code 18.0

2 - The effective length ℓ of the field (normal unit is metres).

3 - The pole tip field B_0 (normal unit is kG).

4 - The pole tip half aperture a. The effective strength of the sextupole is $qB_0\ell/a^2p$.

The first- and second-order transformation elements for a sextupole are given in Ref. 1. In TURTLE these elements are evaluated using the actual momentum of the ray. Therefore, TURTLE can give a very good indication of how effectively chromatic aberrations can be eliminated or minimized in a beam line.

SOLENOID: Type code 19.0

The solenoid is most often used as a focusing element in systems passing low-energy particles. Particles in a solenoidal field travel helical trajectories. The solenoid fringing-field effects necessary to produce the focusing are included.

There are three parameters:

1 - Type code 19.0

2 - Effective length ℓ of the solenoid (metres).

3 - The field (kG). A positive field by convention points in the direction of positive z for positively-charged particles.

BEAM ROTATION: Type code 20.0

A rotation of the coordinates of a ray at a given point in the beam line about the beam axis may be effected by a 20.0 type code. Thus one can simulate a magnet oriented differently than is normally available, or a known rotational misalignment of an element about the beam axis. There are two parameters.

1 - Type code 20.0

2 - The angle of rotation in degrees.

The rotation is taken in a clockwise sense about the beam axis.

HISTOGRAMS: Type codes 50.0, 51.0, 52.0, 53.0

A one- or two-dimensional histogram of beam parameters (listed below) may be requested at any position in the beam line. A one-dimensional histogram is indicated by a 50.0 card. The horizontal coordinate of a two-dimensional histogram is indicated by a 51.0 card and the vertical coordinate by a 52.0 card. The histogram card is placed at the point in the beam line where one wishes to histogram the ray coordinate specified. Any of these three type codes requires five parameters.

1 - Type code 50.n, 51.0 or 52.n, indicating the type of histogram desired. *In DECAY TURTLE, 51.n is also a valid entry.*

2 - A code number for the parameter to be histogrammed (see below).

3 - The lower limit of the histogram.

4 - The upper limit of the histogram.

5 - The interval of the histogram (bin size).

Normally the symbol n is blank or zero. If n is a digit in the range 1 to 9, the histogram is said to be flagged. An entry is made into such a histogram only for rays which have encountered a "flag" element of the form 53.n ; before or after the histogram specification, where n has the same value as on the histogram specification card. If the 53.n ; flag occurs after the histogram specification, the histogram will represent the phase space which can be transmitted through that portion of the beam line falling between the positions of the histogram and its associated flag. *For original TURTLE, the two coordinates of a two-dimensional histogram can refer to only one kind of particle. As such only one flag can be associated with a two-dimensional histogram and must be specified on the 52.n card. Any flag on the 51.n card will be ignored. In decay TURTLE, however, a flag is permitted on both cards, and each of them will refer to the particle specified on that card. An example is given below.*

The meaningful coordinates for the histograms are specified by the following codes:

1: x coordinate
2: x' coordinate

3: y coordinate

4: y' coordinate

6: $\delta = (p - p_0)/p_0$.

In DECAY TURTLE, six more codes are recognized:

8: z, the distance along the reference orbit. Such a histogram will show the distribution of particles which have been stopped by aperture limits.

11: p, the momentum of the particles.

21: $r = \sqrt{x^2 + y^2}$, *the radius.*

22: $XI = \sqrt{x'^2 + y'^2}$, *the emittance angle, the angle the ray makes with respect to the central trajectory.*

23: PSI = arctan (y/x), the polar angle.

24: ZETA = arctan (y'/x').

The codes given above all apply to the parent particles only. In order to specify the charged daughter particles, add 100 to the relevant code. For the neutral daughters, add 200 to the relevant code.

If either of the coordinates specified for a two-dimensional histogram is the distance along the reference orbit (code 8.0), the histogram will refer only to particles lost from the beam. In this case the particle kind specified must be the same on both the 51.0 and 52.0 cards, since it would not make sense to specify different particles on the two cards. Such a loss histogram cannot be flagged. A maximum of 10 such histograms is allowed.

A one-dimensional histogram will be created at any point in a beam line where a 50.0 type code card is inserted. The intervals of the histogram are arranged vertically and specified on the left side of the page. The beam intensity in a given interval is indicated by a horizontal row of the letter "X".

A two-dimensional histogram is actually initiated by the 52.0 card, while the 51.0 card merely supplies a value for the horizontal coordinate. Thus the 51.0 card and the 52.0 card for a two-dimensional histogram need not occur at the same location in the beam line. However, for each 52.0 card there must be one 51.0 card somewhere in the deck preceding it.

Several 52.0 cards can use the same 51.0 card to supply the horizontal coordinate for the histogram. The 51.0 card must simply occur at some point in the deck earlier than the first 52.0 card. Whenever a 52.0 card appears, indicating a vertical coordinate and initiating a histogram, the horizontal coordinate is taken from the 51.0 card most recently preceding it.

For both one- and two-dimensional histograms the coordinates histogrammed with their units and position in the beam line are printed with the histogram. Examples of both one- and two-dimensional histograms are given below.

For a two-dimensional histogram the number of rays falling in a given bin is printed directly for 1-9 rays, represented by a letter A-Z for 10-35 rays, and represented by a $ for more than 35 rays. The $ was chosen because it is a fairly dark symbol. Sums of the rows and columns are given along the edges of the histogram. The numbers giving the sums of columns are themselves to be read vertically. Overflow in all four directions is given below the histogram.

A total of 100 histograms in a given beam line is allowed. The number of intervals allowed per histogram is limited directly only for the horizontal coordinate of a two-dimensional histogram, the limit being 100. If the limits and interval size specified are such that more than 100 intervals would result, the program readjusts the upper limit so that the number of intervals equals 100. There is a limit of 10,000 total locations provided for histogram storage. If N is the number of intervals used in a one-dimensional histogram, the number of storage locations used is N + 3. If NA and ND are the number of intervals used for the horizontal and vertical coordinates respectively of a two-dimensional histogram, the number of locations used is (NA + 1) · (ND + 1) + 5.

Example No. 1

Histograms of the phase-space acceptance and emittance of a beam line

```
1. ......... ;  Initial beam specification

51.   1.  -20.  20.  1.  ;  } Histogram for x,x' phase-space acceptance
52.3  2.  -10.  10.  .5  ;  } passing through beam line and reaching
.                           } flagged position (53.3) at end of beam.
.
.   } Element cards specifying
.   } beam line.
.
.

53.3 } Flag for acceptance histogram (above).
51.   1.  -20.  20.  1.  ;  } Histogram for x,x' phase-space (emittance)
52.   2.  -10.  10.  .5  ;  } at end of beam line.
SENTINEL.
```

Example No. 2

Histograms for a typical decay channel

1. .... ; Initial phase-space (BEAM) specification.

51.1 6. -10. 10. .5 ; — *The $\delta = \Delta p/p_0$ of parents at this location is taken as the horizontal coordinate for the histogram to be displayed at the location of the 52.0 card.*

.
.
.
.
.
.

53.1 — *Only those parents that have passed through this position are used for the histogram.*

.
.

53.2 — *Only those neutral daughters that were generated before this position (i.e. those passing through this position) and which reach the position of the 52.2 card are entered into the histogram.*

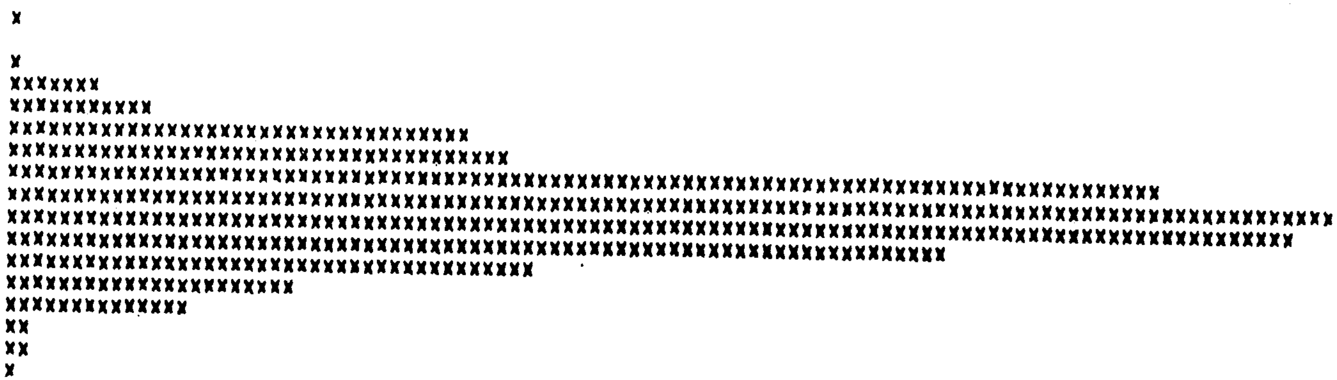
.
.
.
.
.

52.2 211. 0. 40. 1. ; — *The momentum p of the neutral daughter is used as the vertical coordinate for the histogram displayed at this location.*

Since an entry is made in the histogram only if both particles are present, the result is a histogram containing only those particles involved in the decay process between the two flagged positions 53.1 and 53.2. No other 51.0 type code entry should appear between the 51.1 and 52.1 entries, otherwise the flagging process will not function as described above.

Examples of a one-dimensional and a two-dimensional histogram are given in Figs. 3 and 4.

```
HISTOGRAM NC 19
DISTRIBUTION CF    Y' IN MR       FOR MU           580,000 M    FRCM THE TARGET

         INTERVAL                              SCALE FACTCR.. 100 X'S EQUAL    71 ENTRIES
LESS THAN        -2.000          0

  -2,000 TC   -1,900           0
  -1,900 TO   -1,800           0
  -1,800 TO   -1,700           0
  -1,700 TO   -1,600           0
  -1,600 TO   -1,500           0
  -1,500 TO   -1,400           0
  -1,400 TO   -1,300           0
  -1,300 TO   -1,200           0
  -1,200 TO   -1,100           0
  -1,100 TO   -1,000           0
  -1,000 TO    -,900           0
   -,900 TO    -,800           1
   -,800 TO    -,700           0
   -,700 TO    -,600           1
   -,600 TO    -,500           5
   -,500 TO    -,400           8
   -,400 TO    -,300          25
   -,300 TO    -,200          27
   -,200 TO    -,100          62
   -,100 TO     ,000          71
    ,000 TO     ,100          69
    ,100 TO     ,200          81
    ,200 TO     ,300          29
    ,300 TO     ,400          16
    ,400 TO     ,500          10
    ,500 TO     ,600           2
    ,600 TO     ,700           2
    ,700 TO     ,800           1
    ,800 TO     ,900           0
    ,900 TO    1,000           0
   1,000 TO    1,100           0
   1,100 TO    1,200           0
   1,200 TO    1,300           0
   1,300 TO    1,400           0
   1,400 TO    1,500           0
   1,500 TC    1,600           0
   1,600 TO    1,700           0
   1,700 TO    1,800           0
   1,800 TO    1,900           0
   1.900 TO    2,000           0

GREATER THAN     2,000           0

        TOTAL NUMBER CF ENTRIES =      380     INCLUDING UNDERFLOW AND OVERFLOW
        CENTER =   -.013   RMS HALF WIDTH =    .225

HISTOGRAM NO 19
DISTRIBUTION CF    Y' IN MR       FOR MU           580,000 M    FRCM THE TARGET
```



F 3 Example of a histogram for the .antity y'.

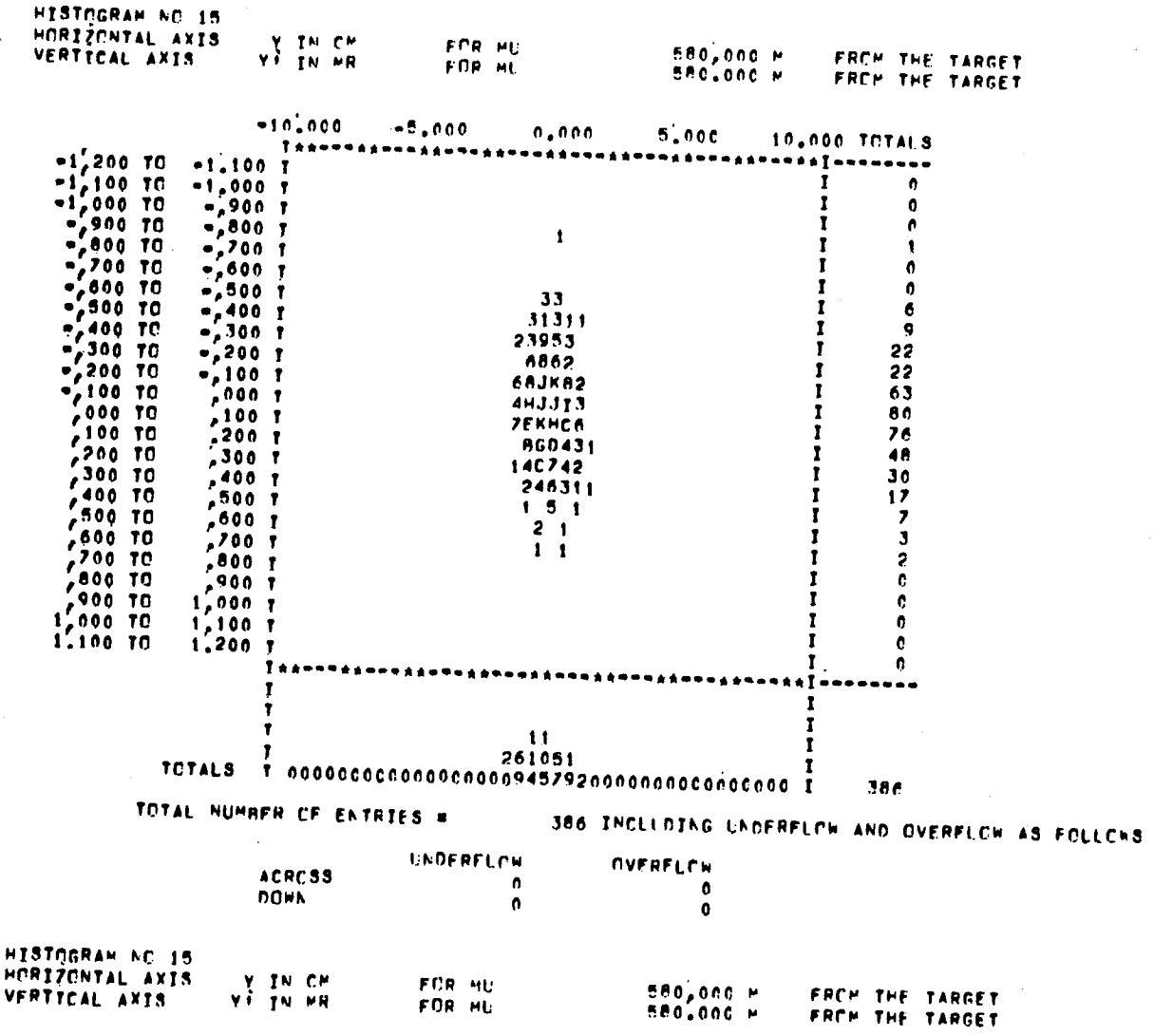


Fig. 4 Example of a two-dimensional histogram of y′ versus y.

DECAY: Type code 60.0

Normally DECAY TURTLE will track parent particles as in the original TURTLE. If decay calculations are to be included, an element with five parameters must be inserted among the data cards, normally after the BEAM cards.

1 - The type code 60.0 specifying decay.
2 - The mass of parent particles (GeV).
3 - The mass of charged daughter particles (GeV).
4 - The mass of neutral daughter particles (GeV).
5 - The parent lifetime in nanoseconds.

The particle masses are given in the same units as the design momentum of the beam line (ignoring the factor c, normally GeV). The unit for mass can be changed by means of a 15. 11. type code entry preceding the 60.0 type code card.

The standard version of decay TURTLE contains the decay kinematics for the two processes

$$\pi \rightarrow \mu + \nu \quad \text{or} \quad K \rightarrow \mu + \nu \ .$$

The particle masses must always be specified; at present no default values are known to the program. A different decay process can be accommodated by changing one or more of the following subroutines:

SUBROUTINE DECAY

generates from the parent RAY the two daughter rays:
RAYC for the charged daughter,
RAYN for the neutral daughter.

SUBROUTINE DECAYP

decodes the 60.0 type code card and prepares data for DECAY.

SUBROUTINE DECAYZ(ZD)

returns the generated random decay position.

The initial phase space of the parent beam has been described in the section on type code 1.0. For decay problems, it is often required to enter a different initial distribution. In such a case the user has to change the SUBROUTINE EMIT which generates a new parent ray at each call.

Users who wish to alter one of the indicated routines are referred to the comments in the main program which describe the significance of all COMMON variables used in the program.

4. HOW TO ACCESS DECAY TURTLE AT CERN

The standard version of decay TURTLE is stored as a 6400 permanent file. If no changes are required, it may be called by the simple sequence

```
job,T100
ACCOUNT(name,div,number)
FIND(TURTLE,TURTLEMPSISELIN,ID=MPSISELIN)
TURTLE.
end-of-record
data cards
end-of-file
```

The FORTRAN deck of the program can be obtained from the UPDATE library stored on TURTLELIBMPSISELIN,ID=MPSISELIN. A typical call deck might be

```
job,T100
ACCOUNT(name,div,number)
FIND(OLDPL,TURTLELIBMPSISELIN,ID=MPSISELIN)
UPDATE.
FTN(I=COMPILE)
REWIND(LGO)
FIND(TEMP,TURTLEMPSISELIN,ID=MPSISELIN)
COPYL(TEMP,LGO,TURTLE)
TURTLE.
end-of-record
update directives to modify the program
end-of-record
data cards
end-of-file
```